Neutrino masses from Planck scale

Takashi Toma
Kyoto University

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In collaboration with Alejandro Ibarra and Patrick Strobl (TUM)

京都大学
KYOTO UNIVERSITY

Introduction

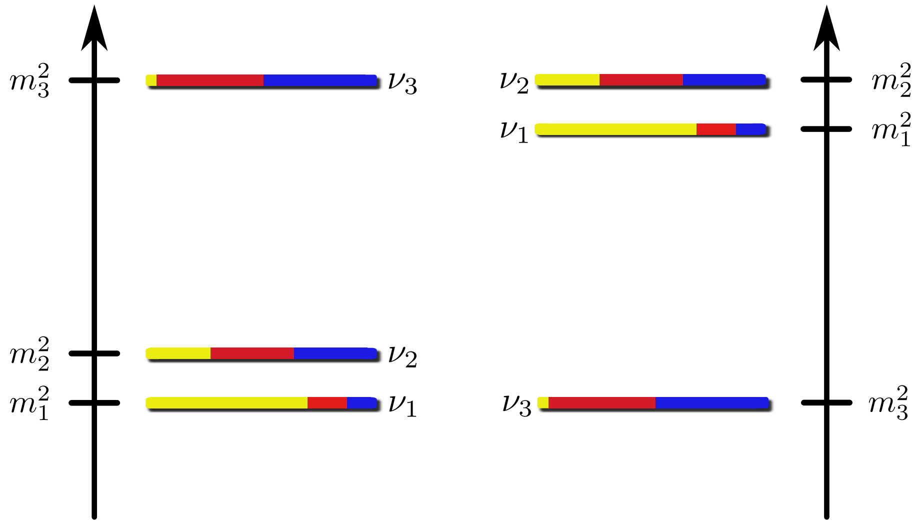
Neutrinos are massive. (massless in the Standard Model)

- Neutrino oscillation data $\Rightarrow \mathcal{O}(0.1)$ eV

	NH	IH
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.306^{+0.012}_{-0.012}$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.587^{+0.020}_{-0.024}$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.02179^{+0.00076}_{-0.00076}$
Δm^2_{21} [eV^2]	$7.50^{+0.19}_{-0.17}\times 10^{-5}$	$7.50^{+0.19}_{-0.17}\times 10^{-5}$
$\Delta m^2_{3\ell}$ [eV^2]	$2.524^{+0.039}_{-0.040}\times 10^{-3}$	$-2.514^{+0.038}_{-0.041}\times 10^{-3}$

Esteban et al. JHEP (2017)

- Very small masses of neutrinos and large mixing angles.
- Mild hierarchy of two heaviest masses $\lesssim 6$.
- $\Rightarrow$ different mechanism of mass generation?



Seesaw mechanism

- There are many neutrino mass generation mechanisms.
- Seesaw mechanism (Type I, Type II, Type III...)
  In Type I seesaw (simplest), three heavy right-handed neutrinos N_R are introduced.

$$\begin{aligned}\mathcal{L} &= -\phi^\dagger \overline{\ell_L} y_\nu N_R - \frac{1}{2}\overline{N_R^c} M N_R + \text{h.c.} \\ &\to -\overline{\nu_L} m_D N_R - \frac{1}{2}\overline{N_R^c} M N_R + \text{h.c.} \qquad m_D = y_\nu \langle\phi\rangle\end{aligned}$$

Mass matrix

$$\begin{array}{cc} \nu_L & N_R^c \end{array}$$

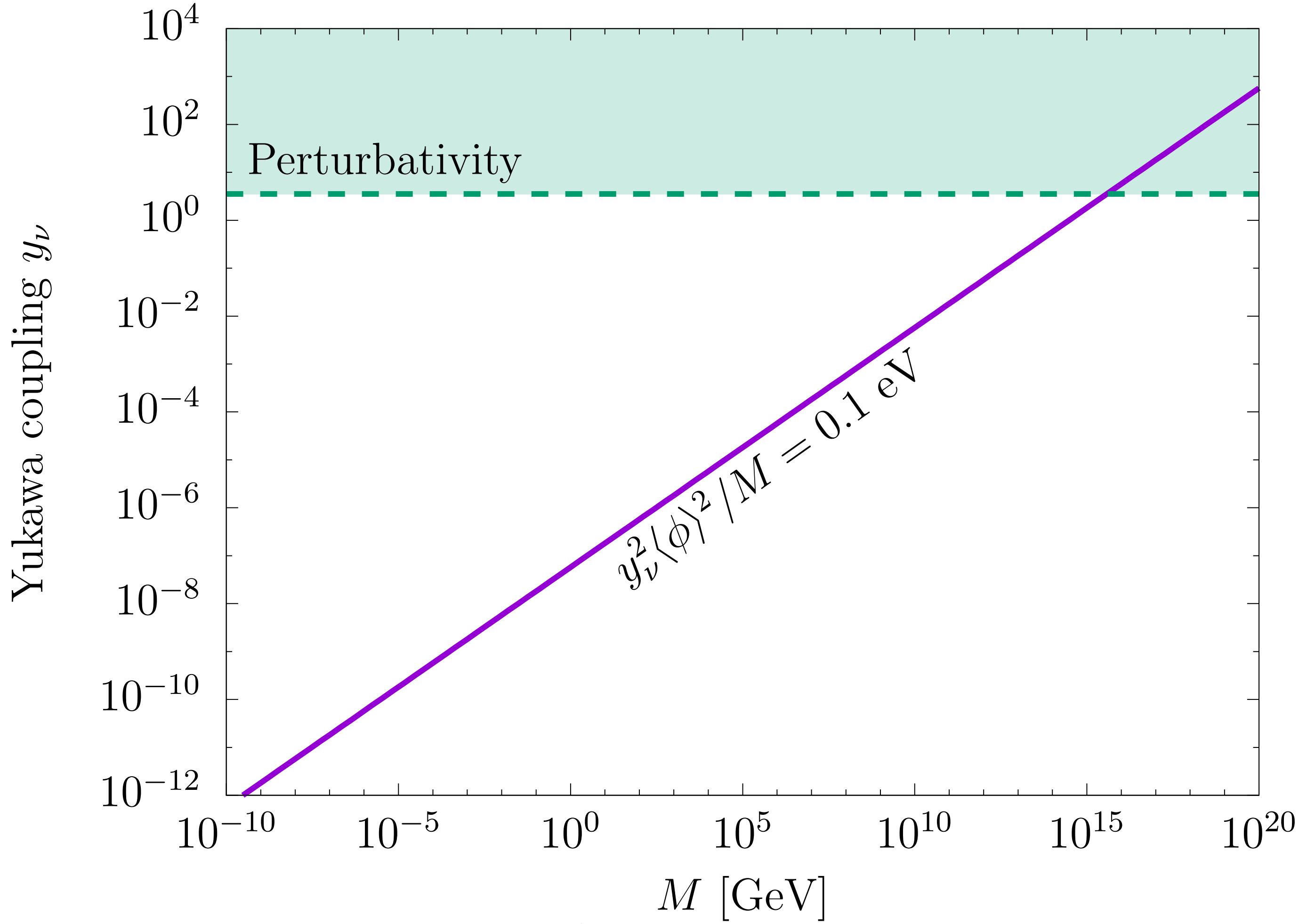
$$\begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \to \quad m_\nu \approx -m_D M^{-1} m_D^T + \cdots$$

(if $m_D \ll M$)

- Rough picture

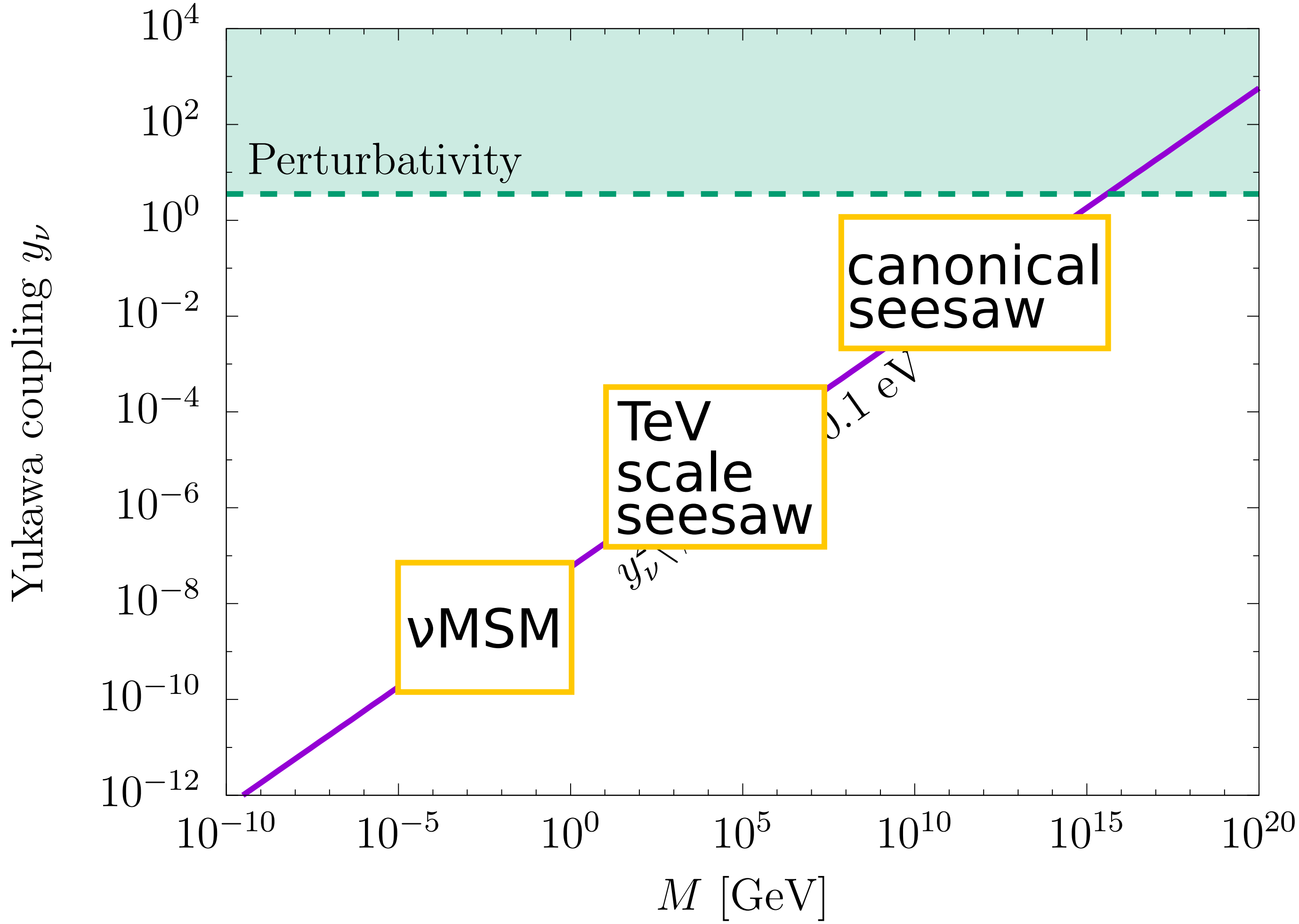
$$m_\nu \sim \frac{y_\nu^2 \langle\phi\rangle^2}{M} \sim 0.1\ \text{eV}$$

Seesaw mechanism



- Ex.1: $y_\nu \sim \mathcal{O}(1)$ for $M = 10^{14}$ GeV
- Ex.2: $y_\nu \sim \mathcal{O}(10^{-7})$ for $M = 1$ GeV

Seesaw mechanism



- An energy scale smaller than Planck scale is necessary.
- Cannot directly correlate neutrino mass scale and Planck scale.

The Model

- Add three right-handed neutrinos.

$$\mathcal{L} = \frac{1}{2}\overline{N_i}\partial\!\!\!/ N_i - \frac{M_{ij}}{2}\overline{N_i^c}N_j - (Y_\nu)_{ij}\,\tilde{H}\overline{L_i}N_j + \mathrm{H.c.}$$

- Assumption: (almost) rank-1 mass matrix at Planck scale.

$$M \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \qquad M = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$$

at Planck scale | at low energy scale

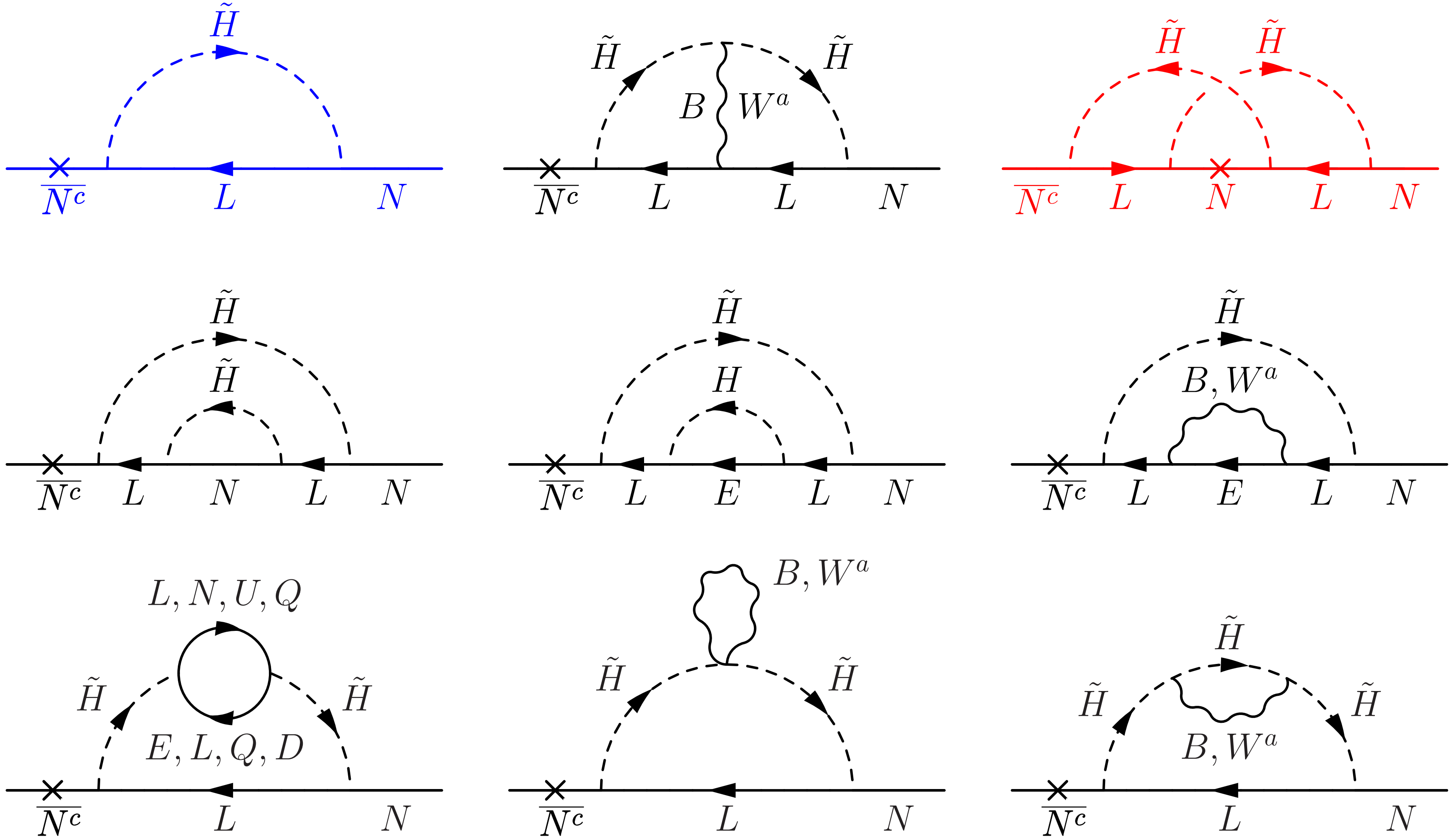
$\rightarrow$ reduce number of parameters

- Right-handed Majorana neutrino masses are expected to be generated via gravitational interactions. $\leftarrow$ No flavor discrimination

$$M = M_0\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \rightarrow \text{Mass eigenvalues} = 0, 0, 3M_0$$

Renormalization Group Equation for M

- M_1 and M_2 are generated by radiative effect.
  $\Rightarrow$ Renormalization group equation (RGE) for M.
- All the diagrams



Renormalization Group Equation for M

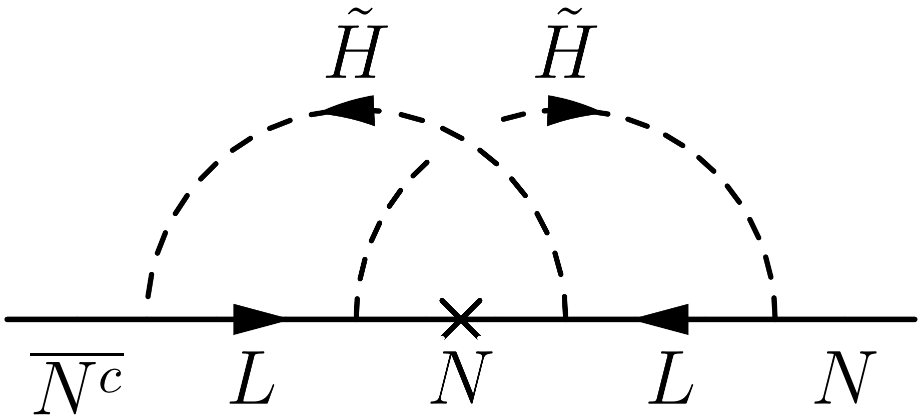
- At 1-loop, only one diagram contributes

$$\beta_M^{\text{1-loop}} = \frac{dM}{dt} = \frac{1}{(4\pi)^2}\left[\left(Y_\nu^\dagger Y_\nu\right)^T M + M\left(Y_\nu^\dagger Y_\nu\right)\right]$$

- At 2-loop, there are many contributions

$$\beta_M^{\text{2-loop}} = \frac{dM}{dt} = \frac{4}{(4\pi)^4}\left(Y_\nu^\dagger Y_\nu\right)^T M\left(Y_\nu^\dagger Y_\nu\right) + \cdots$$

Rank increasing diagram



the other diagrams do not increase rank of M.

Renormalization Group Equation for M

Full beta function

$$\frac{dM}{dt} = \frac{1}{(4\pi)^2}\left[\left(Y_\nu^\dagger Y_\nu\right)^T M + M\left(Y_\nu^\dagger Y_\nu\right)\right] + \frac{4}{(4\pi)^4}\left(Y_\nu^\dagger Y_\nu\right)^T M \left(Y_\nu^\dagger Y_\nu\right)$$
$$+\frac{1}{(4\pi)^4}\left[\frac{17}{8}\left(g_Y^2 + g_2^2\right)\left(Y_\nu^\dagger Y_\nu\right) - \frac{1}{4}Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - \frac{1}{4}Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu\right.$$
$$\left.-\frac{3}{2}\mathrm{Tr}\left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right)\left(Y_\nu^\dagger Y_\nu\right)\right]^T M$$
$$+\frac{1}{(4\pi)^4}M\left[\frac{17}{8}\left(g_Y^2 + g_2^2\right)\left(Y_\nu^\dagger Y_\nu\right) - \frac{1}{4}Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu - \frac{1}{4}Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu\right.$$
$$\left.-\frac{3}{2}\mathrm{Tr}\left(Y_e^\dagger Y_e + Y_\nu^\dagger Y_\nu + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d\right)\left(Y_\nu^\dagger Y_\nu\right)\right]$$

- We include only M and Y_ν.
  The other contributions do not increase rank of M.

Analytic solutions

- Iterative integration: $\dfrac{dM}{dt} = P^T M + MP + 4P^T MP, \quad \left[P \equiv \dfrac{Y_\nu^\dagger Y_\nu}{(4\pi)^2}\right]$

$$\to M(\mu) \approx M(M_P) + \left[P^T M(M_P) + M(M_P)P + 4P^T M(M_P)P\right] \times \log\left(\frac{\mu}{M_P}\right)$$

- Diagnalize $M(\mu)$
- For $\mathrm{Rank}\, M = 1$ case ($M = \mathrm{diag}(0, 0, M_3)$ at Planck scale)

$$M_3(\mu) \approx M_3,$$

$$M_2(\mu) \approx -4M_3 P_{32}^2 \log\left(\frac{M_P}{\mu}\right) \qquad \to 10^{14}\ \mathrm{GeV}\ \text{for}\ Y_\nu = \mathcal{O}(1)$$

$$M_1(\mu) \approx 8M_3 P_{21}^2 P_{32}^2 \log^2\left(\frac{M_P}{\mu}\right) \qquad \to 10^{9}\ \mathrm{GeV}\ \text{for}\ Y_\nu = \mathcal{O}(1)$$

$\to M_1(\mu)$ is comparable to four-loop order

Analytic solutions

- For $\mathrm{Rank}\, M = 3$ case ($M = \mathrm{diag}(M_1, M_2, M_3)$ at Planck scale) assumption: $M_1 \sim M_2 \ll M_3$

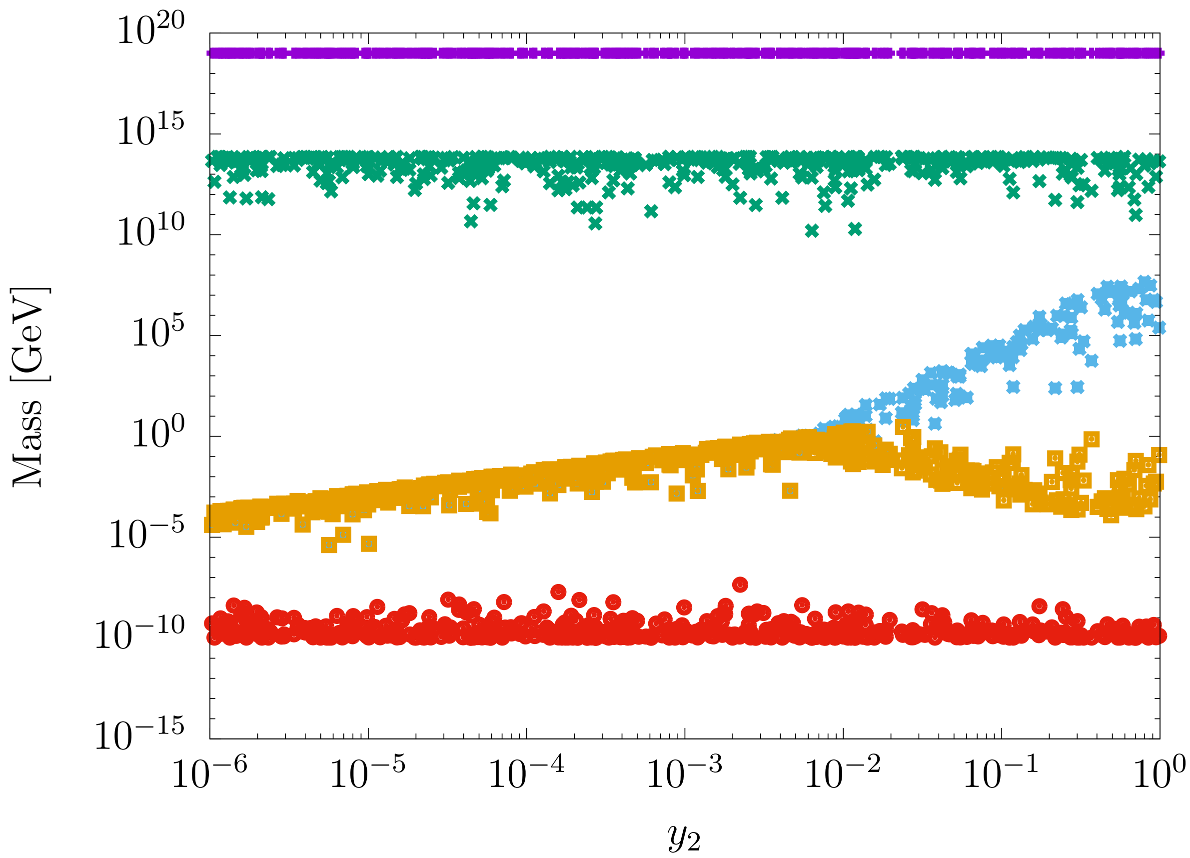
$$
\begin{aligned}
M_3(\mu) &\approx M_3, \\
M_2(\mu) &\approx -4M_3 \left(P_{31}^2 + P_{32}^2\right) \log\left(\frac{M_P}{\mu}\right), \\
M_1(\mu) &\approx 8M_3 \frac{\left[P_{31}P_{32}\left(P_{11} - P_{22}\right) - P_{21}\left(P_{31}^2 - P_{32}^2\right)\right]^2}{P_{31}^2 + P_{32}^2} \log^2\left(\frac{M_P}{\mu}\right)
\end{aligned}
$$

- $M_1 = M_2 = 0 \quad \Rightarrow \quad \mathrm{Rank}\, M = 1$ case is recovered.

- If tree contribution M_1, M_2 is larger than loop induced mass,

$$
M_1(\mu) \approx \frac{M_2 P_{31}^2 + M_1 P_{32}^2}{P_{31}^2 + P_{32}^2}
$$

Numerical analysis ($\mathrm{Rank}\, M = 1$, $\mathrm{Rank}\, Y_\nu = 2$)



- Parametrization

$$Y_\nu = V_L Y_D V_R^\dagger \to Y_D V_R^\dagger$$
$$P = \frac{1}{(4\pi)^2} V_R Y_D^2 V_R^\dagger$$

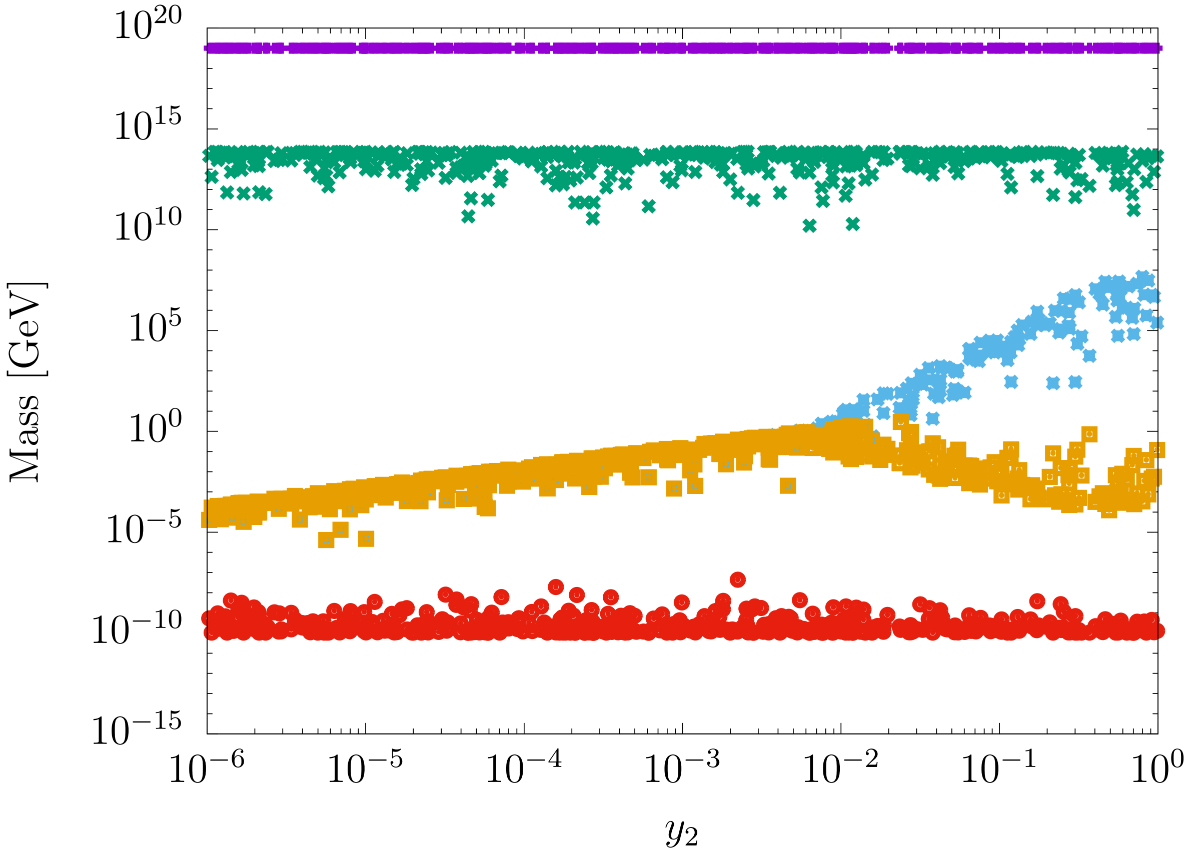
- At Planck scale

$$M = \mathrm{diag}\,(0, 0, M_P)$$
$$Y_D = \mathrm{diag}\,(0, y_2, 1)$$

- Run RGE

- Diagonalize 6×6 mass matix $\begin{pmatrix} 0 & Y_\nu v \\ Y_\nu^T v & M \end{pmatrix}$ at low energy scale
- Note: if $\mathrm{Rank}\, Y_\nu = 1$ at Planck scale, N_1 mass is not generated.

Numerical analysis ($\mathrm{Rank}\, M = 1$, $\mathrm{Rank}\, Y_\nu = 2$)



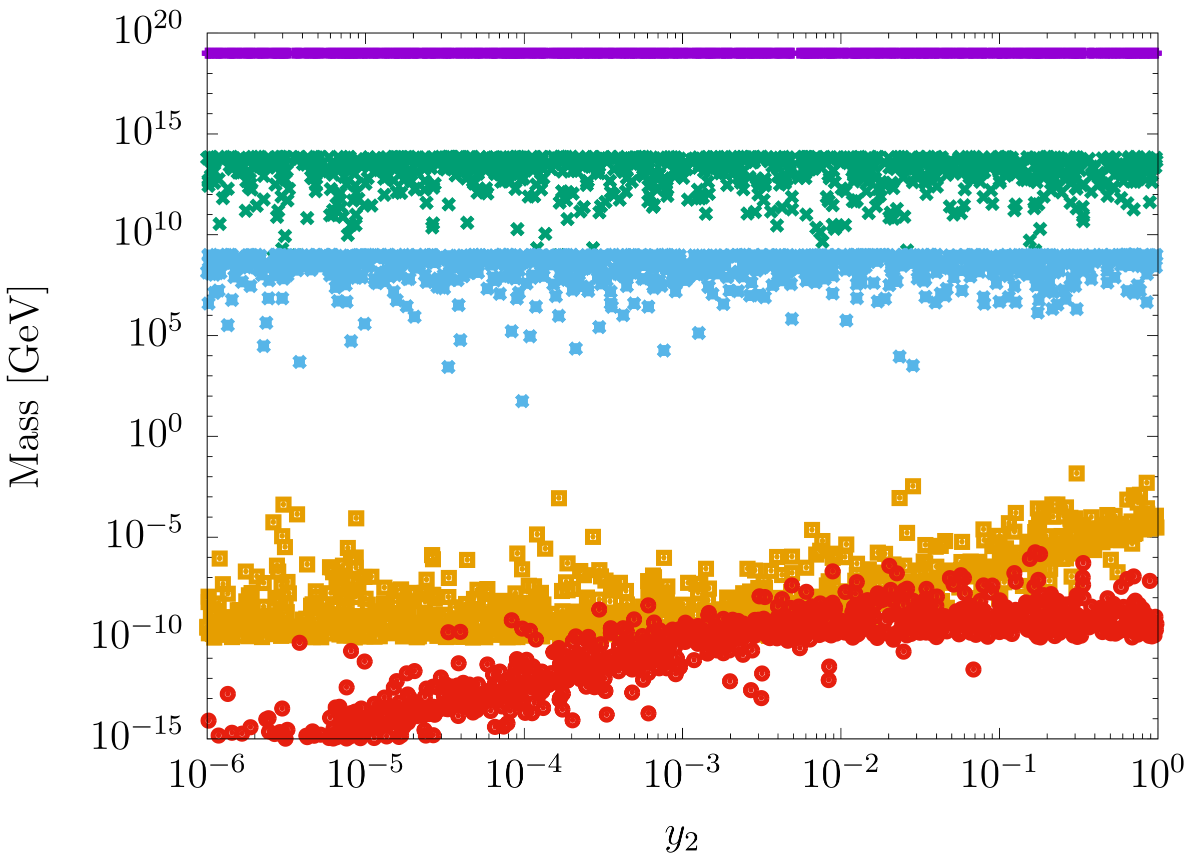
- Heaviest: $M_3|_\mu \sim M_P$
- 2nd heaviest:

$$M_2|_\mu \sim \frac{y_3^4 M_P}{(4\pi)^4} \times \text{mixing}^4 \sim 10^{14}\ \mathrm{GeV}$$

- Lightest: $m_1|_\mu = 0$ because $\mathrm{Rank}\ \left(Y_D M^{-1} Y_D^T\right) = 2$

- 2nd lightest state (red) is always $\sim 0.1\ \mathrm{eV}$.
- The other two states: $y_2 v \pm M_1|_\mu$
  Pseudo Dirac state is constructed by (ν_1, N_1) if $y_2 \lesssim 10^{-2}$.
- cannot generate mild hierarchy

Numerical analysis (Rank $M = 2$, Rank $Y_\nu = 2$)



- At Planck scale

$$M = \mathrm{diag}\,(0, M_2, M_P)$$
$$Y_D = \mathrm{diag}\,(0, y_2, 1)$$
$$M_2 = 10^9\ \mathrm{GeV}$$

- Heaviest, 2nd heaviest, lightest are same with Rank $M = 1$ case.

- Mild hierarchy of small neutrino masses can be obtained if $10^{-4} \lesssim y_2 \lesssim 10^{-2}$

Summary

1. If right-handed neutrino masses are very hierarchical at Planck scale, radiative corrections dominate right-handed neutrino masses at low energy scale.
2. Without any new energy scale, one of small neutrino masses is naturally generated via seesaw mechanism.

Future Works

1. More detailed analysis.
2. This framework leads predictive phenomenology because of reduced number of parameters.
   Ex. application to leptogenesis, the other scenarios with hierarchical mass spectrum.